

All Integration Techniques

Worksheet #2

$$1. \int \cos(2x) dx$$
$$\frac{1}{2} \sin(2x) + C$$

$$2. \int \frac{3x^2 - 5x}{2x\sqrt{x}} dx$$

$$\int \frac{3x^2 - 5x}{2x^{3/2}} dx$$

$$\frac{1}{2} \int (3x^{1/2} - 5x^{-1/2}) dx$$

$$\frac{1}{2} \int 3x^{1/2} - 5x^{-1/2} dx$$

$$\frac{1}{2} [2x^{3/2} - 10x^{1/2}] + C$$

$$3. \int 3x(x+5)^2 dx$$

$$\int 3x(x^2 + 10x + 25) dx$$

$$\int 3x^3 + 30x^2 + 75x dx$$

$$\frac{3}{4}x^4 + 10x^3 + \frac{75}{2}x^2 + C$$

$$4. \int \frac{-3}{\sqrt{2x+7}} dx$$

$$u = 2x + 7$$
$$du = 2 dx$$

$$\int \frac{-3}{u^{1/2}} \cdot \frac{du}{2}$$

$$-\frac{3}{2} \int u^{-1/2} du$$

$$-3u^{1/2} + C$$

$$-3(2x+7)^{1/2} + C$$

$$5. \int 3x \sqrt{2x+5} dx$$

$$u = 2x+5 \quad \int 3x \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2 \quad \frac{3}{2} \int \left(\frac{u-5}{2} \right) u^{1/2} du$$

$$dx = \frac{du}{2} \quad \frac{3}{4} \int u^{3/2} - 5u^{1/2} du$$

$$\frac{3}{4} \left[\frac{2}{5} u^{5/2} - \frac{10}{3} u^{3/2} \right] + C$$

$$\frac{3}{4} \left[\frac{2}{5} (2x+5)^{5/2} - \frac{10}{3} (2x+5)^{3/2} \right] + C$$

$$6. \int \sin^3(3x) \cos(3x) dx$$

$$u = \sin(3x) \quad \int u^3 \cdot \cos(3x) \cdot \frac{du}{3\cos(3x)}$$

$$\frac{du}{dx} = 3\cos(3x) \quad \frac{1}{3} \int u^3 du$$

$$dx = \frac{du}{3\cos(3x)} \quad \frac{1}{12} u^4 + C$$

$$\frac{1}{12} \sin^4(3x) + C$$

$$7. \int x^3 \cos(2x) dx$$

	u	dv
+	x^3	$\cos(2x)$
-	$3x^2$	$\frac{1}{2} \sin(2x)$
+	$6x$	$-\frac{1}{4} \cos(2x)$
-	6	$-\frac{1}{8} \sin(2x)$
+	0	$\frac{1}{16} \cos(2x)$

$$\frac{1}{2} x^3 \sin(2x) + \frac{3}{4} x^2 \cos(2x) - \frac{3}{4} x \sin(2x) - \frac{3}{8} \cos(2x) + C$$

$$8. \int 5^{\cos(x)+2} \sin x dx$$

$$u = \cos(x)+2 \quad \int 5^u \cdot \cancel{\sin x} \cdot \frac{du}{-\sin x}$$

$$\frac{du}{dx} = -\sin(x) \quad - \int 5^u du$$

$$dx = \frac{du}{-\sin(x)} \quad - \frac{5^u}{\ln 5} + C$$

$$- \frac{5^{\cos(x)+2}}{\ln 5} + C$$

$$9. \int 2x(2x+5)^4 dx$$

$$u = 2x+5 \quad \int 2x \cdot u^4 \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2 \quad \int \left(\frac{u-5}{2}\right) u^4 du$$

$$dx = \frac{du}{2} \quad \frac{1}{2} \int u^5 - 5u^4 du$$

$$\frac{1}{2} \left[\frac{1}{6} u^6 - u^5 \right] + C$$

$$\frac{1}{2} \left[\frac{1}{6} (2x+5)^6 - (2x+5)^5 \right] + C$$

$$10. \int \frac{2}{x^2-3x-10} dx \rightarrow \frac{A}{(x-5)} + \frac{B}{(x+2)}$$

$$2 = A(x+2) + B(x-5)$$

$$x = -2 :$$

$$x = 5 :$$

$$2 = B(-7)$$

$$2 = A(7)$$

$$B = -\frac{2}{7}$$

$$A = \frac{2}{7}$$

$$\int \frac{\frac{2}{7}}{(x-5)} + \frac{-\frac{2}{7}}{(x+2)} dx$$

$$\frac{2}{7} \ln|x-5| - \frac{2}{7} \ln|x+2| + C$$

$$11. \int \frac{2x}{x-1} dx$$

$$u = x-1 \quad \int \frac{2x}{u} \cdot du$$

$$\frac{du}{dx} = 1 \quad 2 \int (u+1)u^{-1} du$$

$$du = dx \quad 2 \int 1 + u^{-1} du$$

$$2[u + \ln|u|] + C$$

$$2[(x-1) + \ln|x-1|] + C$$

$$12. \int x^4 e^{3x} dx$$

	u	dv
+	x^4	e^{3x}
-	$4x^3$	$\frac{1}{3}e^{3x}$
+	$12x^2$	$\frac{1}{9}e^{3x}$
-	$24x$	$\frac{1}{27}e^{3x}$
+	24	$\frac{1}{81}e^{3x}$
-	0	$\frac{1}{243}e^{3x}$

$$\frac{1}{3}x^4 e^{3x} - \frac{4}{9}x^3 e^{3x} + \frac{12}{27}x^2 e^{3x} - \frac{24}{81}x e^{3x} + \frac{24}{243}e^{3x} + C$$

$$13. \int e^{3x} \sin(5x) dx$$

$$u = e^{3x} \quad \int dv = \int \sin(5x) dx$$

$$\frac{du}{dx} = 3e^{3x} \quad v = -\frac{1}{5}\cos(5x)$$

$$du = 3e^{3x} dx$$

$$-\frac{1}{5}e^{3x}\cos(5x) + \frac{3}{5} \int e^{3x}\cos(5x) dx$$

$$u = e^{3x} \quad \int dv = \int \cos(5x) dx$$

$$\frac{du}{dx} = 3e^{3x} \quad v = \frac{1}{5}\sin(5x)$$

$$du = 3e^{3x} dx$$

$$\frac{1}{5}e^{3x}\sin(5x) - \frac{3}{5} \int e^{3x}\sin(5x) dx$$

$$\int e^{3x}\sin(5x) dx = -\frac{1}{5}e^{3x}\cos(5x) + \frac{3}{25}e^{3x}\sin(5x) - \frac{9}{25} \int e^{3x}\sin(5x) dx$$

$$\frac{34}{25} \int e^{3x}\sin(5x) dx = -\frac{1}{5}e^{3x}\cos(5x) + \frac{3}{25}e^{3x}\sin(5x)$$

$$\int e^{3x}\sin(5x) dx = \frac{25}{34} \left[-\frac{1}{5}e^{3x}\cos(5x) + \frac{3}{25}e^{3x}\sin(5x) \right] + C$$

$$14. \int_0^3 -4x e^{3x^2} + 2 dx$$

$$\int_0^3 -4x e^{3x^2} dx + \int_0^3 2 dx$$

$$u = 3x^2 \quad \int_0^{27} -4x e^u \cdot \frac{du}{6x} \quad 2(3-0)$$

$$\frac{du}{dx} = 6x \quad -\frac{2}{3} \int_0^{27} e^u du \quad 6$$

$$dx = \frac{du}{6x} \quad -\frac{2}{3} e^u \Big|_0^{27}$$

$$-\frac{2}{3} [e^{27} - e^0]$$

$$-\frac{2}{3} [e^{27} - 1] + 6$$

$$15. \int_0^3 \frac{x-2}{x^2+7x+6} dx \rightarrow \frac{A}{(x+6)} + \frac{B}{(x+1)}$$

$$x-2 = A(x+1) + B(x+6)$$

$$x = -1:$$

$$x = -6:$$

$$-3 = B(5)$$

$$-8 = A(-5)$$

$$B = -\frac{3}{5}$$

$$A = \frac{8}{5}$$

$$\int_0^3 \frac{\frac{8}{5}}{x+6} + \frac{-\frac{3}{5}}{x+1} dx$$

$$\frac{8}{5} \ln|x+6| - \frac{3}{5} \ln|x+1| \Big|_0^3$$

$$\left[\frac{8}{5} \ln 9 - \frac{3}{5} \ln 4 \right] - \left[\frac{8}{5} \ln 6 - \frac{3}{5} \ln 1 \right]$$

$$\frac{8}{5} \ln 9 - \frac{3}{5} \ln 4 - \frac{8}{5} \ln 6$$

$$16. \int_0^{\frac{\pi}{2}} \sec^2 x \tan^2 x dx$$

$$u = \tan x \quad \int_0^{\frac{\pi}{2}} \sec^2 x \cdot u^2 \cdot \frac{du}{\sec^2 x}$$

$$\frac{du}{dx} = \sec^2 x \quad \int_0^1 u^2 du$$

$$dx = \frac{du}{\sec^2 x}$$

* undefined upper bound

$$17. \int x^2 \ln(2x) dx$$

$$u = \ln(2x)$$

$$\int dv = \int x^2 dx$$

$$\frac{du}{dx} = \frac{2}{2x}$$

$$v = \frac{1}{3}x^3$$

$$du = \frac{1}{x} dx$$

$$\frac{1}{3}x^3 \ln(2x) - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$$

$$\frac{1}{3}x^3 \ln(2x) - \frac{1}{3} \int x^2 dx$$

$$\frac{1}{3}x^3 \ln(2x) - \frac{1}{9}x^3 + C$$

$$18. \int_0^1 \frac{4x^2}{\sqrt[3]{2x-3}} dx$$

$$u = 2x-3$$

$$\int_{-3}^{-1} \frac{4x^2}{u^{1/3}} \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2$$

$$2 \int_{-3}^{-1} \left(\frac{u+3}{2}\right)^2 u^{-1/3} du$$

$$dx = \frac{du}{2}$$

$$\frac{1}{2} \int_{-3}^{-1} (u^2 + 6u + 9) u^{-1/3} du$$

$$\frac{1}{2} \int_{-3}^{-1} u^{5/3} + 6u^{2/3} + 9u^{-1/3} du$$

$$\frac{1}{2} \left[\frac{3}{8} u^{8/3} + \frac{18}{5} u^{5/3} + \frac{27}{2} u^{2/3} \right]_{-3}^{-1}$$

$$\frac{1}{2} \left[\left[\frac{3}{8} + \frac{-18}{5} + \frac{27}{2} \right] - \left[\frac{3}{8}(-3)^{8/3} + \frac{18}{5}(-3)^{5/3} + \frac{27}{2}(-3)^{2/3} \right] \right]$$

$$19. \int_0^2 x^2 \sqrt{3x-5} dx$$

$$u = 3x-5$$

$$\int_{-5}^1 x^2 \cdot u^{1/2} \cdot \frac{du}{3}$$

$$\frac{du}{dx} = 3$$

$$\frac{1}{3} \int_{-5}^1 \left(\frac{u+5}{3}\right)^2 u^{1/2} du$$

$$dx = \frac{du}{3}$$

$$\frac{1}{27} \int_{-5}^1 (u^2 + 10u + 25) u^{1/2} du$$

$$\frac{1}{27} \int_{-5}^1 u^{5/2} + 10u^{3/2} + 25u^{1/2} du$$

$$\frac{1}{27} \left[\frac{2}{7} u^{7/2} + 4 u^{5/2} + \frac{50}{3} u^{3/2} \right]_{-5}^1$$

$$20. \int_0^{\frac{\pi}{2}} x^2 \cos(3x) dx$$

	u	dv
+	x^2	$\cos(3x)$
-	$2x$	$\frac{1}{3} \sin(3x)$
+	2	$-\frac{1}{9} \cos(3x)$
-	0	$-\frac{1}{27} \sin(3x)$

$$\frac{1}{3} x^2 \sin(3x) + \frac{2}{9} x \cos(3x) - \frac{2}{27} \sin(3x) \Big|_0^{\frac{\pi}{2}}$$

$$21. \int_1^5 dx$$

$$1(5-1)$$

$$4$$